Problem: Derive the Inverse Square Law applied to solar radiation, starting with the rate at which the sun emits radiative energy and the radius of the sun. Although this problem doesn't ask for a numerical solution, adapt the strategy outlined in the handout "Good Physical Problem Solving Strategy" (which applies more fittingly to problems that ask for a quantitative solution, unlike this one).

I. Information Given or Otherwise Known

- A known "reference" distance from the center of the sun $\equiv r_{ref}$
- Rate at which the sun emits radiative energy $\equiv ER_{sun}$

II. Information Desired

- A relation between intensity (that is, flux) of solar radiative energy evaluated at any arbitrary distance from the sun $\equiv S(r)$, and the solar radiation energy flux evaluated at some other known reference distance $\equiv S(r_{ref})$

III. Relations Needed

(1) $F = R/A$

[Generic relation between a flux $\equiv F$, a rate $\equiv R$, and a surface area $\equiv A$. The context is one in which some quantity encounters, passes through, is absorbed by, reflects from, or is emitted from a surface of some sort.]

(2) $A(r) = 4\pi r^2$

[Relation between the surface area of a sphere $\equiv A(r)$ and the radius of the sphere $\equiv r$.]

IV. Solution

(A) Apply Eq. (1) to radiative energy emitted from the surface of the sun:

$$(3)\ S(r_{sun}) = ER_{sun}/A(r_{sun})$$

where $r_{sun} \equiv$ is the distance from the center of the sun to the sun's surface (that is, the radius of the sun), $S(r_{sun}) \equiv$ solar radiation (emission) flux at the surface of the sun (that is, "evaluated at distance $= r_{sun}$ from the center of the sun"), and $A(r_{sun}) \equiv$ area of a sphere of radius $= r_{sun}$ (that is, the surface area of the sun).

(B) We will assume that solar radiative energy is conserved as it propagates away from the sun. That is, we'll ignore the effects of the small amount of matter in the solar system surrounding the sun, which can absorb solar radiation, reflect it back toward the sun, and emit its own radiative energy. Under this assumption, then the rate at which solar radiative energy passes through (is emitted from) the sun's surface should be the same as the rate at which it passes through the surface of progressively larger (imaginary) spheres farther and farther from the sun. (That is, none of it is destroyed or transformed into other forms of energy, ,and no new solar radiation is created or added as it propagates away from the sun.)

If this is true, then we can apply Eq. (1) to solar radiative energy passing through a sphere of any arbitrary radius $\equiv r$:

$$(4)\ S(r) = ER_{sun}/A(r)$$

where $S(r) \equiv$ solar radiative flux at distance $= r$ from the center of the sun, and $A(r) \equiv$ area of the sphere of radius $= r$ through which the solar radiation is passing. (ER_{sun} is the same in Eq. (4) and Eq. (3), since we've assumed that solar radiative energy is conserved as it passes through progressively larger (imaginary) spheres.

(C) Solve Eq. (3) for ER_{sun} and substitute the result into Eq. (4):

$$(5)\ S(r) = S(r_{sun}) \times A(r_{sun})/A(r)$$

(D) Apply Eq. (2) to the sphere represented by the sun and to the sphere of arbitrary radius = r and substitute the results into Eq. (5):

$$(6)\ S(r) = S(r_{sun}) \times (4\pi\, r_{sun}^{2}) / (4\pi\, r^{2})$$

$$= S(r_{sun}) \times r_{sun}^{2}/r^{2} \qquad (4\pi \text{ cancels out})$$

$$S(r) = S(r_{sun}) \times (r_{sun}/r)^{2} \qquad (\text{ratio of squares = square of the ratio})$$

(E) Eq. (6) constitutes a particular, somewhat restricted version of the Inverse Square Law applied to solar radiation. The relation says that if we know the intensity of solar radiation (solar radiation flux) at the sun's surface ($S(r_{sun})$, where it is an emission flux) and if we know the radius of the sun (r_{sun}), then we can determine the intensity of solar radiation (solar radiation flux, $S(r)$) at any other distance from the sun (r).

However, in Step (A) if we had applied Eq. (1) to any other known distance from the sun (call it r_{ref}, for "reference distance) where we happen to know the solar radiation flux ($S(r_{ref})$), then we would end up with the same relation as in Eq. (6) except that r_{ref} replaces r_{sun} and $S(r_{ref})$ replaces $S(r_{sun})$, and we have a more general version of the Inverse Square Law:

$$(7)\ S(r) = S(r_{ref}) \times (r_{ref}/r)^{2}$$

V. Check Solution

- The units make sense because the units of distance in r_{ref} and r cancel out in the ratio, leaving only units of $S(r_{ref})$ on the right-hand side, which are units of energy flux, as desired.

- $S(r_{ref})$ is always a positive (that is, > 0) flux so $S(r)$ also has to be positive (as we expect).

- The relation makes intuitive sense because as r increases, the right-hand side decreases (in proportion to the reciprocal of r squared, and hence the "inverse square" law), so the solar radiation flux (left-hand side) evaluated at the increasing distance (r) must also decrease. This occurs because as solar radiation propagates away from the sun, the same amount of energy passes through spheres with larger and larger surface areas, so the same amount of solar energy "spreads out" over larger and larger surface areas and hence becomes progressively less concentrated on surfaces (that is, less intense).

(Note that this mechanism of spreading out of radiative energy differs from the kind of spreading out that occurs when radiative energy strikes a surface at an angle less than $90°$. In this derivation of the inverse square law, the solar radiation always strikes perpendicular the surface of progressively larger spheres.)