Evaluating Feedback in Unbalanced (as Well as Balanced) Budget Systems

- In a budget system, the (net) rate at which some quantity, *Q*, changes with respect to time equals the difference between the sum of the sources of *Q* and the sum of the sinks of *Q*. (This budget equation expresses a conservation law for the quantity *Q*. For example, *Q* could, among many other physical properties, be the heat content of a bit of matter.)
- If a *source or sink of Q depends on Q*, then there is *feedback* in the budget system.
- Suppose there is such feedback in the budget system for *Q*. If there is a non-zero (net) rate of change in *Q* (that is, the sources and sinks don't balance), then *Q* will change (at a certain rate). As *Q* changes, so does one of the sources or sinks. As the source or sink changes, the *difference* between the sum of sources and sum of sinks of *Q* changes, which means that *the net rate of change of Q changes* (as the budget equation tells us).
- If the net rate of change of *Q* becomes smaller (that is, the change in *Q* leads to a *slowing* of the net rate of change of *Q*), then the feedback is *negative*. The feedback *damps* the rate of change in *Q*.
- If the net rate of change in *Q* becomes larger (that is, the change in *Q* leads to a faster (net) rate of change in *Q*), then the feedback is *positive*. The feedback *amplifies* the rate of change in *Q*.

A Generic Budget Equation for Q

($Q \equiv$ physical property of a bit of matter)

$$\Delta Q/\Delta t = \sum \text{sources of } Q - \sum \text{sinks of } Q$$

This equation says that the rate at which property Q of an object (a chunk of matter) changes with time ($\Delta Q/\Delta t$) equals the difference between (a) the sum of rates at which the object gains Q by various physical mechanisms and (b) the sum of rates at which the object loses Q by various mechanisms.

If one of the rates at which the object gains or loses Q itself depends on Q, then as Q changes, so will that rate of gain or loss, and hence so will the difference between the sums of rates of gain and loss, and hence, so will $\Delta Q/\Delta t$. This constitutes ***feedback*** in the budget system for Q.

If $\Delta Q/\Delta t$ becomes smaller as a result of the feedback, then the feedback is ***negative*** (and Q changes more slowly as a result of changes in Q—that is, the feedback reduces the budgetary imbalance and ***damps*** the rate of change).

If $\Delta Q/\Delta t$ becomes larger as a result of the feedback, the feedback is ***positive*** (and Q changes more rapidly as a result of changes in Q—that is, the feedback increases the budgetary imbalance and ***amplifies*** the rate of change).

Heat Budget Equation (Conservation of Energy) for Earth

$\Delta H / \Delta t = \sum$ sources of $H - \sum$ sinks of H

$= $ solar absorption rate (SR_{abs}) – LWIR emission rate (ER_{IR})

$= (SR_{arriv} - SR_{refl}) - EF_{IR} \times A_{sphere}$

$= [SR_{arriv} - (\alpha \times SR_{arriv})] - (\sigma T^4) \times As_{phere}$

$= SR_{arriving} \times (1 - \alpha_E) - \sigma T^4 \times (4\pi R_E^2)$

$= (SF(r_{E\text{-}s}) \times A_{Xsect}) \times (1 - \alpha_E) - \sigma T^4 \times 4\pi R_E^2$

$= SF(r_{E\text{-}s}) \times (\pi R_E^2) \times (1 - \alpha_E) - \sigma T^4 \times 4\pi R_E^2$

$= [SF(\bar{r}_{E\text{-}s}) \times \left(\frac{\bar{r}_{E-s}}{r_{E-s}}\right)^2] \times \pi R_E^2 \times (1 - \alpha_E) - \sigma T^4 \times 4\pi R_E^2$

Temperature Equation for Earth

(1) $\Delta H / \Delta t = SF(\bar{r}_{E\text{-}s}) \times \left(\frac{\bar{r}_{E-s}}{r_{E-s}}\right)^2 \times \pi R_E^2 \times (1 - \alpha_E) - \sigma T^4 \times 4\pi R_E^2$

(2) $\Delta H = c_H m \, \Delta T$

Substitute Eq. (2) into Eq. (1) and solve for $\Delta T / \Delta t$:

$$\Delta T / \Delta t = \frac{1}{c_H m} \times \left\{ SF(\bar{r}_{E-s}) \times \left(\frac{\bar{r}_{E-s}}{r_{E-s}}\right)^2 \times \pi R_E^2 \times (1 - \alpha_E) - \sigma T^4 \times 4\pi R_E^2 \right\}$$

Temperature Equation for Earth's Surface

Include additional sources and sinks:

$$\Delta T/\Delta t = \frac{1}{c_H m} \times \{SR_{abs} + LWIRRa_{bs} \pm ConR - LHER \ - ERI_R\}$$