ERTH 535: Planetary Climate Change | **The Effective Radiative Temperature of a Planet** | **Dept. of Earth & Climate Sci, SFSU**

Problem: Determine the effective radiating temperature of a planet, given its albedo and distance from the sun. Assume that the planet's heat budget is approximately balanced.

Start with a statement of a balanced heat budget for the planet as a whole (from Conservation of Energy), and introduce other physical and geometric relations as needed to connect the information desired to the information given or known. One way to show the connections among these relations is as follows:

(1) $$0 \cong SR_{abs} - ER$$

(6) $$ER = EF \times A_{sphere}$$

(8) $$A_{sphere} = 4\pi R^2$$

(2) $$\begin{aligned} SR_{abs} &= SR_{arriving} - SR_{reflected} \\ &= SR_{arriving} - \alpha \times SR_{arriving} \\ &= SR_{arriving} \times (1-\alpha) \end{aligned}$$

(7) $$EF = \sigma T^4$$

(3) $$SR_{arriving} = SF(r_{p2s}) \times A_{X\sec t}$$

(5) $$A_{X\sec t} = \pi R^2$$

(4) $$SF(r_{p2s}) = SF(\overline{r}_{p2s}) \times \left(\frac{\overline{r}_{p2s}}{r_{p2s}}\right)^2$$

Equation (1) is the statement of a balanced heat budget for the planet as a whole, where:

- $SR_{abs} \equiv$ rate at which planet absorbs solar radiative energy
- $ER \equiv$ rate at which planet emits radiative energy to space

Equation (2) relates the rate at which the planet absorbs solar radiation to the rate at which solar radiation strikes the planet and to the planet's albedo, where:

- $SR_{arriving} \equiv$ rate at which solar radiative energy strikes (arrives at) a planet
- $SR_{reflected} \equiv$ rate at which solar radiative energy reflects from a planet
- $\alpha \equiv$ albedo of a planet $\equiv SR_{reflected} / SR_{arriving}$

Equation (3) is the relation between a rate and a flux, applied to solar radiation striking the planet, where:

- $r_{p2s} \equiv$ the planet's distance from the sun
- $SF\left(r_{p2s}\right) \equiv$ the flux of solar radiation at the top of the planet's atmosphere, directly facing the sun
- $A_{X\sec t} \equiv$ the area of a cross-section through the center of the planet (the effective area that the planet presents to the sun)

Equation (4) is the Inverse Square Law, in which the planet's solar constant serves as the reference value, where:

- $\bar{r}_{p2s} \equiv$ the mean distance of the planet from the sun
- $SF\left(\bar{r}_{p2s}\right) \equiv$ the flux of solar radiation at the top of the planet's atmosphere directly facing the sun, at distance = $\bar{r}_{p2s}$ from sun

Equation (5) relates the area of a circle to its radius, where:

- $R_p \equiv$ radius of the planet

Equation (6) is the relation between a rate and a flux, applied to the planet's emission of radiative energy, where:

- $EF \equiv$ the planet's radiative emission flux
- $A_{sphere} \equiv$ the surface area of the planet (a sphere)

Equation (7) is the Stefan-Boltzmann Law, where:

- $T \equiv$ effective radiating temperature of a planet

Equation (8) relates the surface area of a sphere (A_{sphere}) to the radius of the sphere (R).

To find the effective radiating temperature of the planet, first make all of the substitutions suggested by the arrows to get:

$$0 \cong SF\left(\bar{r}_{p2s}\right)\times\left(\frac{\bar{r}_{p2s}}{r_{p2s}}\right)^2 \times \pi R_p^2 \times (1-\alpha) - \sigma T^4 \times 4\pi R_p^2$$

Now, divide the entire equation above by πR_p^2:

$$0 \cong SF\left(\bar{r}_{p2s}\right)\times\left(\frac{\bar{r}_{p2s}}{r_{p2s}}\right)^2 \times (1-\alpha) - \sigma T^4 \times 4$$

Solve for T^4:

$$T^4 \cong \frac{SF\left(\bar{r}_{p2s}\right)\times\left(\frac{\bar{r}_{p2s}}{r_{p2s}}\right)^2 \times (1-\alpha)}{4\sigma}$$

Solve for T by taking the fourth root of both sides:

$$T \cong \left(\frac{SF\left(\bar{r}_{p2s}\right)\times\left(\frac{\bar{r}_{p2s}}{r_{p2s}}\right)^2 \times (1-\alpha)}{4\sigma}\right)^{1/4}$$

This resembles the expression derived in Chapter 3 of our text. In our expression here, $SF\left(\bar{r}_{p2s}\right)\times\left(\frac{\bar{r}_{p2s}}{r_{p2s}}\right)^2$ is the flux of solar radiation at the top of a planet's atmosphere, facing the sun (which is given a single symbol in the text).

Using this relation, if we know a planet's distance from the sun (r_{p2s}), the planet's albedo (α), the solar constant for the planet ($SF\left(\bar{r}_{p2s}\right)$), and the planet's mean distance from the sun ($\bar{r}_{p2s}$), then we can determine the effective radiating temperature of any planet in our solar system.