Dr. Dave Dempsey
Dept. of Earth & Climate Sciences, SFSU

Problems #1: Total, Material, and Local Derivatives

Due Friday, Oct. 27
(10 points)

Solve the two problems below, following the strategic and formatting guidelines provided. (These guidelines are based on research on how good quantitative problem-solvers organize their approach to solving such problems.) You'll be providing a structured, multi-section, step-by-step presentation of your solutions. Your numerical answers constitute only a small part of the credit for this assignment (10%); the coherence and clarity of your presentation (following the guidelines) constitutes the rest.

(1) A heavily loaded ship steams westward at a rate of 10 km/hr. The surface pressure varies with respect to horizontal position in that direction by 3.5 Pa/km. What is the pressure tendency (that is, the rate at which pressure varies with respect to time at a fixed point) as recorded by a barometer on a nearby island, if the pressure aboard the ship is observed to decrease at a rate of 100 Pascals per 3 hours? (Express your answer in millibars/3 hrs. There are 100 Pascals (Pa) in one millibar (mb). Treat 3 hours as a time unit, like we treat 1 hour.)

Instructions

(A) You don't need to restate the problem, since this assignment does that for you. Instead, first create a section with the header:

I. <u>Information Given or Otherwise Known</u>

Identify each piece of quantitative and qualitative information provided in the problem statement that you ultimately use to solve the problem, and summarize them in a bulleted list. Present the quantitative information using the following format:

- *English description/name* ≡ *shorthand symbol* = *numerical value and units* [× *conversion factor(s)* = *converted value and units*]

(The square brackets in this format indicate that you might or might not need to convert the units of a quantity. If you do, you wouldn't include the square brackets—they aren't part of the format when you apply it.)

Note the use of "$\equiv$" and "=" here. The symbol "$\equiv$" means "is equivalent to", or "is defined to mean", or "is alternatively expressed as". In contrast, "=" above means "has the quantitative value of", which has quite a different meaning. Of course, we use "=" in mathematical equations, too, where it means "is quantitatively equal to", even when we write it using only symbols.

For your shorthand symbols, you can in principle use whatever you want because you are explicitly defining what they mean in this section. However, for clarity of communication with your audience it's best to try to follow established conventions when they exist (conventions used in this class would be a good place to start). Consider using simple mnemonic symbols when conventions don't exist or if you're not sure what the conventions might be.

For this particular problem, two examples might look like this:

- Horizontal speed of ship $\equiv c_H$ = 10 km/hr
- Ship's direction of motion is westward

<u>Symbols</u>. In ERTH 430 we've been using the symbol $\vec{c}$ to represent the velocity of things other than fluid parcels. Velocity is a three-dimensional vector, but in this problem we're given only the ship's horizontal speed (which is the magnitude of the horizontal part, or component, of the ship's velocity in some coordinate systems). The symbol c (without the arrow over it) is one convention for representing the magnitude of the vector $\vec{c}$. The magnitude of $\vec{c}$ (that is, c) is the full three-dimensional speed, one of the velocity vector's components in certain coordinate systems. Attaching the subscript "H" to the symbol c gives us c_H, which the bulleted statement above defines to represent the horizontal speed of the ship, which is the magnitude of the horizontal part of the ship's velocity. The problem doesn't give us explicit information about the vertical component of the ship's velocity, unless we interpret the statement that the ship is steaming westward to mean that it is moving horizontally only. However, the lack of explicit information about the ship's vertical component of velocity might have been an oversight, so we'll probably have to make an assumption about

it. (The ship could be rising and falling significantly as it steams through sea swell, for example.)

Unit conversion. It turns out there is no need to convert the units for the ship's horizontal speed in this problem, a fact that becomes clear in hindsight. Sometimes you don't know in advance whether or not you'll have to convert the units, but if you do, then you can come back to this first section and add the conversion as needed. In some problems the *direction* of an object's motion needs to be represented using a symbol and treated quantitatively (for example, as an angle in a coordinate system). In this problem, if you're careful about your choice of coordinate system, then that won't be necessary, and the example provided above treats it in essence as qualitative information (so no symbol or numerical value is assigned to it).

Numerical values. Convert relatively large and small numbers (those smaller than 0.1 or 0.01 and larger than 10 or 100) to scientific notation (that is, using powers of ten).

Information to list. Don't list information that you don't ultimately use to solve the problem, even if it is provided in the problem description.

(B) Now create a second section with the header:

II. Information Desired

Identify the piece or pieces of information that you're asked to find in the problem, and list it (or them) in a bulleted list using this format:

English description/name $\equiv$ *shorthand symbol*

(C) Create a third section with the header:

III. Relations Needed

The previous two sections are essentially bookkeeping sections to identify clearly what you know and what you want to know in the problem. I like to say that 95% of the creative challenge of solving a problem is in this third section, where you identify the mathematical relation(s) that connect what you know to what you want to know.

As an aside, I distinguish between two types of relations: (a) physical relations, which express (mathematically) physical principles or laws derived empirically or theoretically, and (b) mathematical relations, which are more abstract and don't express a physical principle or law (but might still apply to the physical world). Examples of physical relations include the ideal gas law, any conservation law, the Stefan-Boltzmann relation (between a black body radiator's intensity of radiative emission and its absolute temperature), and the relation between changes in an object's heat content and changes in its temperature. Some examples of mathematical relations include the Pythagorean theorem, the relation between radius, angle and arc length, the relation between total and partial derivatives, the Fundamental theorem of calculus, and the product rule of calculus.

Regardless of whether a relation is physical or purely mathematical, if you need it (to connect the information you know to the information that you want to know), then you should list it in Section III. Use this format:

(#) *relation* (*name or description of the relation*)

An example might look like this:

(1) $\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g_0$ (Vertical equation of motion, assuming no friction)

(where p is pressure, w is the vertical component of a fluid parcel's velocity, and ρ is the fluid density).

The number assigned to the equation (here it's (1)) allows you to refer to the equation later in your solution presentation, e.g., "Eq. (1)".

If symbols appear in the relation that weren't already defined in sections I or II, you should define them. In this example, three (presumably new) symbols, w, p, and ρ, are defined, but not g_0, the global-average acceleration of gravity at Earth's surface. That's because g_0 is a commonly known quantity (that is, not given in the problem statement but "otherwise known") and hence would be listed in Section I. Note that z and the short-hand symbols for derivatives in the example above are not defined because they are so conventionally understood.)

What relation(s) do you need to solve the problem? This is where the art of problem solving comes in. Ultimately, "relations needed" must connect (a) the information that you're given or otherwise know in the problem to (b) the information that you want to know. It follows that *at least one of the relations* that you identify and list in Section III *must include the quantity that you want to know* (listed in Section II).

Moreover, to get a solution, you must ultimately relate the information that you want to know to information that you know (listed in Section I) and *no other (unknown) information*. That is, if your final solution includes information that you don't know (is not listed in Section I), then it's not a solution. Hence, good candidate relations typically include quantities that you know (listed in Section I).

(When you list a relation in Section III that includes quantities listed in Section I and/or II, *use the same symbol*.[1] *Don't define multiple symbols for the same thing*.)

Sometimes an otherwise good candidate relation includes an unknown quantity. In that case, you have several possible tactics:

1. Identify another relation that connects the unknown quantity to quantities that you know. (This additional relation might itself include another unknown quantity, so you might need to find yet another relation that connects the new unknown to known quantities. This could in principle continue through a whole series of relations.)
2. Make a physically reasonable (justifiable) assumption that allows you to ignore the unknown quantity because it's influence is arguably negligible or its value is approximately zero, or ignore the difference between the unknown quantity and another, similar quantity. (You might be able to use *qualitative* information listed in Section I to justify such an assumption.) For example, you might assume that the ship in Problem **(1)** doesn't rise or fall significantly, or that the pressure tendency on the nearby island is about the same throughout the area, including where the ship is located.

[1] There is an exception to this guidance. In some cases you might want to find several values of the same quantity, but a relation you need is the same for all of them. Rather than list the same relation several times using different symbols (for example, using different subscripts on the relevant variables), you should list the relation in Section III once, using more general (say, unsubscripted) variable names. You can then develop a general solution and "apply" it to each specific case in the problem by attaching case-appropriate subscripts to the more generic variables.

3. If you can't ignore a quantity that you don't know, you might be able to approximate it using information that you do know. For example, you might be able to approximate a derivative using finite differences.

Sometimes you can invoke a simplifying assumption as a note attached to the affected relation listed in Section III, making it clear what the assumption is, its impact on the relation, and what the justification is if you can. (In other cases, simplifying assumptions and approximations are better presented as part of the step-by-step solution that you narrate in the next section.)

(D) Create a fourth section with the header:

IV. <u>Solution</u>

In this section you'll describe your solution step by step (ideally labeling the step (say, with capital letters) and narrating clearly what you're doing, in two stages. First you'll manipulate the relation(s) listed in Section III (step by step) to develop a symbolic relation that includes the quantity that you want to know (listed in Section II) on the left-hand side and only quantities that you know (listed in Section I) on the right-hand side, with a box around the whole thing to highlight it:

desired quantity = expression involving only known quantities

(As part of your step-by-step, narrated solution, you'll eliminate any "undesired" unknown quantities appearing in the relations by substituting for them using relations you've already identified for that purpose and listed in Section III.)

Next you'll substitute from Section I the numerical values and units of the known quantities into the right-hand side of the symbolic solution. Having done that, you'll proceed as follows, step by step:

1. For each separate term in the solution involving two or more multiplicative factors, group the units together, any powers of ten together, and the remaining numerical values together, separately.
2. Simplify the units (canceling as appropriate).
3. Simplify the powers of 10.

4. Calculate a single numerical value from the grouped numerical values. (If necessary, convert the units of the final solution to those desired.)

Highlight the final, numerical solution by putting a box around it. For example:

p = 1004.3 mb

(E) Create a final section with the following header:

V. <u>Check Solution</u>

In this section you'll check to make sure that your solution is reasonable. You should typically include the following:

(1) Are the units correct? (That is, are they units for the type of quantity you've solved for? For example, if you've solved for a pressure, are the units actually units of pressure? And if the problem statement asked for particulate units, is the final solution expressed in those units?)
(2) Is the sign right? (For example, if the desired quantity is by nature always non-negative, does it in fact have a positive sign?)
(3) Is the magnitude reasonable? (This check is typically the hardest because you must have some intuition about the typical range of magnitudes that occur in the real world, and compare your answer to that range, or better, to the range you might expect in the particular context of your problem.)

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(2) The next day, the just-unloaded ship, riding noticeably higher on the water, steams eastward at a rate of 28 km/hr. The surface pressure everywhere in the region is lower than it was the day before, but the pattern (shape) hasn't otherwise changed. Aboard the ship, the first mate reads the barometer and observes the pressure decreasing at a rate of 100 Pa in 3 hours. What would the pressure tendency be as recorded by a barometer on the nearby island, expressed in millibars per three hours?

To solve this problem, follow the same guidelines as for Problem **(1)**.