ERTH 430: Fluid Dynamics in Earth Systems	**Solution to a Sample Problem**	Dr. Dave Dempsey Fall 2016

The Problem:

Suppose that the skull of someone you know (not you, of course) has a cranial capacity of 1.4 liters (1400 cm^3), which is about the average volume of a human cranium (though the range is about ±30% around the average). Moreover, suppose that your acquaintance's head is hollow and has a hole in it. Finally, suppose that the atmospheric pressure is 1000 mb, the temperature of the air is at the deep-body temperature (98.6°F), and the air is completely dry.

What is the mass of the air inside your acquaintance's head (in grams)?

Some constants that might (or might not) come in handy:

Gas constant for dry air ≡ R_d = 287 (J/kg)/K

Specific heat of dry air at constant pressure ≡ C_p = 1004 (J/kg)/K

Stefan-Boltzmann constant ≡ σ = 5.67x10^{-8} (W/m^2)/K^4

I. Information Given or Otherwise Known

* Volume of acquaintance's skull $\equiv V = 1400cm^3 \times (1m/100cm)^3$

$$= 1.4 \times 10^3 cm^3 \times (10^{-2}\, m/cm)^3$$

$$= 1.4 \times 10^3 cm^3 \times 10^{-6}\, m^3/cm^3$$

$$= 1.4 \times 10^{-3} m^3$$

* Temperature of air in skull $\equiv T = 98.6^\circ F$

$$= (98.6°F - 32°F) \times \left(\frac{5°C}{9°F}\right) \times \left(\frac{1K}{1°C}\right) + 273.15K$$

$$= 310.15K$$

* Pressure of air in skull $\equiv p = 1000mb \times (100Pa/mb) = 1 \times 10^5 Pa$

* Gas constant for dry air $\equiv R_d = \left[287 (J/kg)/K\right] \times \left(1 Pa \cdot m^3/J\right)$

$$= 287 \left(Pa \cdot m^3/kg\right)/K$$

* Air in the skull is dry

II. Information Desired

* Mass of air in skull $\equiv m$

III. Relations Needed

(1) $p = \rho R_d T$ (Ideal Gas Law for dry air)
where $\rho \equiv$ density of the dry air

(2) $\rho = m/V$ (relation between density, mass, and volume
[or definition of density, $\rho \equiv m/V$?])

IV. Solution

(A) Substitute Eq. (2) into Eq. (1):

(3) $$p = (m/V)R_d T$$

(B) Solve Eq. (3) for *m*:

(4) $$m = pV/(R_d T)$$

(C) Substitute values and units into (4):

$$m = \frac{10^5\, Pa \times 1.4 \times 10^{-3}\, m^3}{\left(2.87 \times 10^2 \left(Pa \cdot m^3/kg\right)/K\right) \times 3.1015 \times 10^2\, K}$$

$$= \left(\frac{1.4}{2.87 \times 3.1015}\right) \times \left(\frac{10^5 \times 10^{-3}}{10^2 \times 10^2}\right)\left(\frac{Pa \cdot m^3}{\left(Pa \cdot m^3/(kg \cdot K)\right) \cdot K}\right)$$

$$= \left(\frac{1.4}{2.87 \times 3.1015}\right) \times 10^{-2}\, kg$$

$$= 0.16 \times 10^{-2}\, kg \times (1000\, gm/kg)$$

$$m = 1.6 \text{ gm}$$

V. Checking the Solution

* Units are in grams, as desired.
* Result is positive, as we'd expect of a mass.
* Magnitude seems reasonable, because the density of air at sea level is typically around 1.2 kg/m^3, so a cubic meter of air at sea level would have about 1.2 kg in it, whereas our friend’s head is on the order of 1000 times smaller volume and so would have on the order of 1000 times less mass of air in it, consistent with what we calculated.