ERTH 430: Fluid Dynamics in Earth Systems

Lab #3: Numerical Modeling of 1-D Shallow Water Waves in a Wave Tank

Dr. Dave Dempsey
Earth & Clim. Sci.
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(Friday, Dec. 8; 20 pts.)

I. Objectives

- Compare observations and numerical simulations of periodically forced, one-dimensional (1-D) waves in a wave tank.
- Explore effects of dissipative vs. reflective boundary conditions on such waves.

II. Materials

- Water tank with wave generator
- Ruler; stop watch; and masking tape
- MATLAB and numerical model of periodically forced, 1-D, shallow water waves.

III. Background: The 1-D Shallow Water Wave Equations

In Lab #2 and in lecture, using (a) qualitative and quantitative observations of periodically forced water waves in a linear wave tank in the classroom; (b) both quantitative and qualitative reasoning (including some hand waving!); and (c) some mathematical manipulation, we have simplified and reduced the general governing equations for fluids to two equations in two unknowns:

$$(1)\quad \frac{\partial u}{\partial t} = [-u\frac{\partial u}{\partial x}] - g_0\frac{\partial h}{\partial x}$$

$$(2)\quad \frac{\partial h}{\partial t} = [-u\frac{\partial h}{\partial x}] - h\frac{\partial u}{\partial x}$$

where x is the coordinate of position measured along the length of the tank from some reference point; and t is time (measured from an arbitrary starting point). These are the *independent variables* in the equations (that is, they don't depend on each other or anything else—we can pick whichever values we want and ask questions about what is going on at that place (x) and time (t)).

There are two *dependent (field) variables* in these questions: $u(x,t)$ is the horizontal component of the water velocity, and $h(x,t)$ is the depth of the water. These are field variables; they depend on location (x) and time (t).

Equation (1) is the *velocity tendency equation* for $u(x,t)$, and Equation (2) is a form of the *continuity equation in tendency form*, transformed into a *tendency equation for the depth of the fluid* ($h(x,t)$).

Each equation has an advection term and a source or sink term for fluid parcels. In the case of Equation (1), the source/sink of $u(x,t)$ (that is, a physical mechanism that can cause $u(x,t)$ of a fluid parcel to change) is the horizontal pressure-gradient force/mass. By vertically integrating the hydrostatic equation (which is itself an approximate form of the vertical velocity equation), we solved for the pressure at any depth in the water and substituted it into the pressure-gradient force/mass term in the horizontal velocity equation to eliminate the pressure from the equations. The horizontal pressure-gradient force/mass term is now expressed in terms of the gradient of the depth of the fluid. This makes sense because under the hydrostatic approximation, the pressure at any depth depends on the depth of water above that level.

These equations are *differential* equations because they involve derivatives. They are *coupled* equations because the equation for each of the two dependent variables requires information about the other one.

The equations are known as the *one-dimensional, shallow water wave equations*, and they apply when the fluid is *frictionless*, *hydrostatic*, *incompressible*, and *spatially one-dimensional* (horizontal in this case). (They are spatially one-dimensional for two reasons: (a) the density and temperature are the same throughout the water (that is, they are *spatially uniform*), so there are no gradients of temperature or density, which, if they were present, would make the horizontal pressure gradient vary with depth and hence create differences in u with depth; and (b) the tank has long, straight sides and the wave generator creates waves the vary mostly along the tank but very little across the tank. Condition (a) removes the vertical variations in u, and condition (b) means that lateral (cross-tank) variations in u are very small compared to variations along the tank, so we ignore them.

The equations are called shallow water equations because they apply under conditions in which the wavelength of the waves is long compared to the depth of the water. When that condition is met, the hydrostatic approximation is reasonable to make. It turns out that in shallow water waves, waves of different wavelength generally travel at the same speed, and that speed = $\sqrt{g_0 \bar{h}}$, where $\bar{h}$ is the average depth of the water. (In water where the depth is comparable to or greater than the wavelength of the waves—that is, "deep water waves"—the speed of the waves depends on their wavelength.)

I've placed the advection terms in Equations (1) and (2) in square brackets to draw attention to the fact that I'm keeping these terms in the equations even though our scaling of the terms suggest that we can neglect them. (That is, for the waves that we've observed in the wave tank, the advection terms are significantly smaller than the other terms that we've kept.) We could in fact neglect them, but they are often kept as part of the shallow water wave equations because they contribute to some interesting, though secondary, behavior of the waves.

Equations (1) and (2) don't have general, exact solutions for a given set of initial conditions (that is, $u(x,0)$ and $h(x,0)$ and boundary conditions (that is, $u(x_b,t)$ and $h(x_b,t)$, where x_b is the coordinate location of a boundary of the water, if there are one or two such boundaries). There are exact solutions for all x and t in very simple cases, but to get solutions more generally we have to solve the equations approximately on a computer. That is, we have to construct a numerical model based on versions of the equations in which we solve the equations at discrete points in space and at discrete times, rather than at all points in space and at all times, and approximate the derivatives in the equations using finite differences between the discrete points in space and in time. As a result, the solutions are not exact, and the errors can be a problem. There is a whole branch of mathematics (numerical analysis) devoted to understanding and minimizing these errors.

In this lab, you'll compare the solutions of a numerical, 1-D, shallow water wave model to observations of waves in our wave tank, and explore the behavior of periodically forced waves when we change the boundary condition from a dissipative boundary (where the energy of the waves is dissipated at the end of the tank and little of that energy reflects back the other direction) to a reflective boundary (where wave energy reflects back the other direction and interacts with the waves coming from the wave generator).

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IV. Instructions

For these experiments, use the same water depth and wave generator period (use frequency setting = 5) as you did in Lab #2. The perforated ramp should be in place at the far end of the tank from the wave generator. Before you turn on the wave generator, *measure the undisturbed water depth ($\bar{h}$).* Then turn on the wave generator and let the wave pattern develop.

(1) [6 pts total] Measurements of wave behavior

(a) [4 pts] *Record measurements of the wave period (T), wavelength (L), and wave half-amplitude (A) of the waves that develop*. (You might have to measure *A* at several places, since it probably varies a bit because although the perforated ramp dissipates much of the energy of waves that encounter it, a little wave energy does still reflect back from the end of the tank and interacts with the waves coming from the wave generator.)

(b) [2 pts] In idealized shallow water waves, the waves travel at a speed $\equiv c = \sqrt{g_0 \bar{h}}$, and there is a relationship between wavelength, wave period, and wave speed of traveling waves: *c* = *L*/*T*. *Using your observations, calculate the wave speed using each of these two relations. Do they agree?* (This is in part a measure of the extent to which the waves behave like shallow water waves, but it could also be a measure of how accurate your measurements are.)

(2) [6 pts] Numerical modeling of waves in the wave tank

On any computer in any of the student computer labs in the Department, start MATLAB and navigate to:

Courses > E430 > Lab3 > *YourName*

where *YourName* is the name of a folder with your name on it. In that folder you should see a file named "sw1d_shell.m". This is a MATLAB program that presents an interface through which you can run a 1-D shallow water wave model, created for use with our wave tank, and plot the results. Run this MATLAB program by typing "sw1d_shell" in the MATLAB Command Window and pressing the "Return" key.

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When you run sw1d_shell, you will be prompted for several key parameters that configure the model run. These include:

- Depth of undisturbed water in the wave tank ($\bar{h}$) in meters.
- Length of the wave tank (in meters).
- Period of forcing by the wave generator, which is also the period of the waves generated (T), in seconds.
- Half-amplitude of the wave generator (that is, half of the back-and-forth displacement of the portion of the wave generator paddle that pushes on the water, averaged over the depth of the water), in meters. (Note: You will need to measure this, or rather calculate it from several measurements. It won't necessarily be the same as the half-amplitude of the waves created by the wave generator.)
- The boundary condition at the far end of the tank from the wave generator:
    - 1 ➔ dissipative boundary (provided by the sloped, perforated ramp), and
    - 2➔ reflective boundary (provided by the detachable solid metal ramp, which you must hold vertically in place as the waves strike it).

For each parameter there is a default value, which you can select simply by pressing the "Return" key. Otherwise you can enter a different desired value and press "Return".

Once you've selected and entered these five parameters, the model will run and output will be stored in files in subfolder "output" in your *YourName* folder. JPEG images at a series of times will be created and stored in subfolder "images", and MATLAB will display these in sequence as an animation. There are two sets of images animated: Figure 1 shows water depth ($h(x,t)$); and then, in a separate window on top of the first plot, Figure 2 shows the horizontal water velocity ($u(x,t)$). (You can drag the Figure 2 window off of the Figure 1 window to see both.)

You might have to click on the Figure 1 or 2 window to coax MATLAB into accelerating the animation, which can sometimes stall otherwise.

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Run MATLAB using parameter values that correspond to those relevant to your observations of waves in the wave tank. Once the model has run and displayed its results, use the Finder on your computer to navigate to the folder where JPEG images of the water depth (file names starting with "h") are stored, in:

Courses > E430 > Lab3 > *YourName* > images

Double-click on one of the images from part way through the simulation, and print a copy. Select another image somewhat farther along in the simulation (but not more than one full wave period later) and print it, too. *On at least one of the hard copies, put your name and the five input values that you specified.*

Using the hard copy images, *estimate the quantities below as simulated by the model. Show your work; annotate the plots as needed.*

- the wavelength (L)
- the wave period (T)
- the wave speed (c)

Compare these values to your observed values of L and T and your value of c calculated from them. How well does the model do?

(3) [8 pts total] <u>Further wave experiments and modeling</u>

(a) [3 pts] Measure the distance from the wave generator paddle to the base of the perforated ramp at the other end of the tank. Instead of using the ramp as a dissipative boundary, you will erect a solid, vertical wall as a reflecting boundary. For this purpose we've designed a piece of plexiglass with notched "arms" attached to it at a right angle. At the base of the ramp, insert the plexiglass wall vertically into the water with its supporting "arms" resting along the top of the sidewalls of the tank. Drop a small vise grip into the notch in each arm and tighten the vise against the side walls to hold the arms in place. Turn on the wave generator and let waves created by the wave generator develop.

- How many wavelengths (L) is the distance from the wave generator paddle to the reflective boundary?
- Qualitatively speaking, describe how the wave pattern that develops differs from what it was with a dissipating boundary, if it differs.
- Measure the amplitude of waves at several locations that you measured them earlier. Have they changed?

(b) [3 pts] Turn off the wave generator, move the reflecting boundary a distance $L/2$ closer to the wave generator paddle, let any waves die down, restart the wave generator, and let the wave pattern develop.

Respond to the same three questions above.

(c) [2 pts] Run numerical model simulations for each of the two cases in **(3)(a)** and **(b)** above. Comment on how well (or not) the model simulates your observations.