ERTH 430:
Fluid Dynamics
in Earth Systems

Lab #1: The Vertical Pressure Gradient Force/Mass in the Atmosphere and Ocean, Part II: Evaluating Hydrostatic Balance

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(Wednesday, Nov. 8; 15 pts)

I. Objectives

- Estimate the vertical pressure gradient force/mass at various altitudes or depths
- Estimate the degree to which the hydrostatic approximation is valid.
- Account for differences in the pressure profiles between atmosphere and ocean and at different altitudes in the atmosphere.

II. Materials

- Radiosonde soundings recorded at three different latitudes: tropics (station SKSP), midlatitude (KOAK), and high latitude (PABR).
- Argo ocean sounding recorded in the southwestern Pacific Ocean.
- Mac laptop computer in the classroom or an iMac in a Department computer lab (TH 518 or TH 607).

III. Instructions and Questions

On one of our Mac computers in the classroom or Departmental computer lab, start MATLAB. Navigate to:
/ > Users > student > Courses > E430 > Lab1 > Atmosphere

(1) (a) [5 pts] Pick one of the sounding data files (your choice) and print a copy of the data. (One way to do this is to access the data using a Web browser at http://funnel.sfsu.edu/CoursesFolder/E430/Lab1/Atmosphere, and if it prompts you for an application to use to open the file, specify TextEdit or another basic text editor. Print the file from there, preferably in landscape mode rather than portrait mode.)

Pick a level near Earth's surface, a second level near the middle of the profile, and a third level high in the profile. At each of these three levels, *estimate (i) the density, (ii) the vertical pressure gradient, and (iii) the vertical pressure gradient force per unit mass* ≡ $(F_{PG})_z/m$*), in MKS units.* Based on your results, *how do the density and the vertical pressure gradient appear to vary with increasing altitude?* (Before you proceed, be sure to read the guidance that follows on the next three pages.)

Part II: Evaluating Hydrostatic Balance

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Since $(F_{PG})_z/m$ is not a quantity that appears in the sounding data files, to estimate it you'll need to relate it to information the does appear in these files. At this point, you know only one relationship involving $(F_{PG})_z/m$, which we developed in class:

$$\text{(A)} \qquad (F_{PG})_z/m = -\frac{1}{\rho}\frac{\partial p(z)}{\partial z}$$

where $z \equiv$ the vertical coordinate of position (in this case, altitude); $p(z) \equiv$ atmospheric pressure expressed as a function of (that is, it depends on) altitude; and $\rho \equiv$ air density.

Unfortunately, neither ρ nor $\partial p/\partial z$ are quantities that appear in the sounding data file, so this relationship by itself isn't enough to allow us to estimate $(F_{PG})_z/m$.

Recall that $\partial p(z)/\partial z \equiv \lim\limits_{\Delta z \to 0} \dfrac{p(z+\Delta z)-p(z)}{(z+\Delta z)-z} = \lim\limits_{\Delta z \to 0} \dfrac{p(z+\Delta z)-p(z)}{\Delta z}$.

Taking a cue from this definition, we can estimate $\partial p(z)/\partial z$ using a finite different approximation (that is, by stopping short of taking the full limit as $\Delta z \to 0$):

$$\partial p(z)/\partial z \equiv \lim_{\Delta z \to 0} \frac{p(z+\Delta z)-p(z)}{\Delta z} \cong \frac{p(z+\Delta z)-p(z)}{\Delta z}$$

Unlike $\partial p/\partial z$, altitudes (z) and pressures (p) are quantities listed in the sounding data file, at discrete levels in the atmosphere. Generally speaking, the smaller Δz is, the more accurate the finite difference approximation will tend to be (and the approximation becomes exact at altitude z in the limit as $\Delta z \to 0$). Hence, to estimate the vertical pressure gradient at each of your three levels of the full sounding, you'll want to use adjacent pairs of altitudes.

For our purposes, it would be more convenient to adopt an alternative way of labeling altitudes. Since we know altitudes at a numbered set of discrete levels, let's write:

$$\text{(B)}\quad \partial p\left(z^*\right)\big/\partial z \cong \frac{p\left(z_{i+1}\right)-p\left(z_i\right)}{z_{i+1}-z_i}$$

where z_i and z_{i+1} are particular (adjacent) altitudes; the subscript *i* represents any of the values in the column of sounding data labeled "LEV" ("SFC" for Earth's surface, "1" for the first level of sounding data above the surface, "2" for the second level of sounding data above the surface, etc.); and z^* is an ambiguous level somewhere between the adjacent levels z_{i+1} and z_i. In fact, it can be shown that $\left(p\left(z_{i+1}\right)-p\left(z_i\right)\right)\big/\left(z_{i+1}-z_i\right)$ equals (exactly) the *average* vertical pressure gradient between the levels z_i and z_{i+1} (though the average is actually defined differently, using an integral and invoking the Fundamental Theorem of calculus).

This leaves the density, ρ. Fortunately, we developed a version of the ideal gas law in class that relates air density to air pressure, air temperature, and the mixing ratio of water vapor in the air, and all of the latter three quantities are present in the sounding files. (We also know the value of the gas constant for dry air, R_d, in the relationship.)

We could substitute the approximation (B) and the expression for ρ (from the ideal gas law) and into the equation (A) for $\left(F_{PG}\right)_z\big/m$, but this creates a problem: from what level between z_i and z_{i+1} should we use values of temperature, pressure, and water vapor mixing ratio to replace the density, given that $\left(p\left(z_{i+1}\right)-p\left(z_i\right)\right)\big/\left(z_{i+1}-z_i\right)$ equals the *average* value of the vertical pressure gradient between levels z_i and z_{i+1}

? If z_i and z_{i+1} are adjacent levels, then we know the temperature, pressure, and mixing ratio only at z_i and z_{i+1}, not between them.

The best we can do is either:

(*i*) estimate an average density, $\bar{\rho}$, in the layer between levels z_i and z_{i+1} using the information available to us:

$$\bar{\rho} \equiv 0.5 \times \left(\rho\left(z_{i+1}\right) + \rho\left(z_i\right) \right)$$

where $\rho\left(z_i\right)$ and $\rho\left(z_{i+1}\right)$ are the densities at levels z_i and z_{i+1} calculated from the ideal gas law; or

(*ii*) estimate an average temperature ($\bar{T}$), average pressure ($\bar{p}$), and average mixing ratio ($\bar{w}$) for the layer (using the information available to us at the two adjacent levels, z_i and z_{i+1}), and use these estimated average values to calculate an estimated average density.

Approaches (*i*) and (*ii*) won't generally give quite the same values for an average density in the layer between levels z_i and z_{i+1} (and neither gives us the actual average density, which is defined using an integral over the layer), but the two estimates will typically be pretty close to each other. You should pick one.

(b) [2 pts] If we ignore friction in the vertical direction in each sounding, *how large would you estimate the vertical acceleration to be at each of your three levels? How large is it as a percentage of the force/mass of gravity? How large would you expect this percentage to be if the hydrostatic approximation were good? Explain. Would you say that the hydrostatic approximation is relatively good at your three levels?*

(c) [1 pt] If the hydrostatic approximation is valid based on your calculations (at least for your sounding), *explain why the rate at which* *the pressure decreases with increasing altitude (that is, the vertical pressure gradient, $\partial p(z)/\partial z$) must itself decrease with increasing altitude.*

Now, navigate to Lab1 > Ocean.

(2) (a) [4 pts] Print a copy of the Argo sounding data file ("OcnSnding.dat", which you can access using a Web browser at http://funnel.sfsu.edu/CoursesFolder/E430/Lab1/Ocean/OcnSnding.dat).

Pick a level near the ocean surface, one near the middle of the profile, and one deep in the profile. *At each level, estimate the density, the vertical pressure gradient, and the vertical pressure gradient force per unit mass* (PGF/mass, in MKS units). Show your work. Based on your results, *how do the density and the vertical pressure gradient appear to vary with increasing depth?* (Before you proceed, read the guidance below.)

As in the calculation of vertical PGF/mass for the atmosphere, you will need to know the density, in this case of seawater. The equation of state for sea water does not have a simple form based on theory like the ideal gas law; instead, it is an empirical relationship created by fitting a combination of polynomials to careful and detailed measurements of pressure, temperature, salinity, and density. For our purposes, it is enough to rely on one of a number of equation of state calculators for seawater that are available online, such as one from the Applied Physics Laboratory at Johns Hopkins University: http://fermi.jhuapl.edu/denscalc.html.

When narrating your calculation of the PGF/mass, be clear about the values that you entered into the online calculator and the values that it returned.

(b) [2 pts] If we ignore any friction that might be acting on seawater in the vertical direction in the sounding, *how large would you estimate the vertical acceleration to be at each of your three levels?* [Note: Be sure to get the sign of the force/mass of gravity correct, given that the convention adopted for the vertical coordinate is positive downward.] *How large is the vertical acceleration as a percentage of the force/mass of gravity? Would you say that the hydrostatic approximation is relatively good at your three levels? Explain.*

(c) [1 pt] *If the hydrostatic approximation is a good approximation based on your calculations (at least for your sounding), explain why the rate at which the pressure varies with increasing depth (that is, the vertical pressure gradient) hardly varies at all.*