In-Class Exercise:
Total and Local Derivatives, the Gradient, and a Relation among Them

1. Refer to Figure 1. Temperatures at a particular date and time are labeled on a regular grid of points in a 2-D field of temperature. An observer with a thermometer is located at location (3 km, 3 km) in the rectangular coordinate system shown. What temperature does the observer record?
2. Suppose that by the time shown in Figure 2, the observer traveled to (5 km, 1 km). During that time, on the average how rapidly did s/he observe temperature to change? How rapidly did temperature change at the observer's starting point? What about the ending point?
3. Suppose that another observer started in Figure 1 at (5 km, 3 km) and by the time shown in Figure 2 had moved to (3 km, 5 km). How rapidly did s/he observe temperature to change, on the average? How rapidly did the temperature change at the observer's starting location, on the average? Does there seem to be a discrepancy between what the observer observes and what happens at the observer's starting location? (What about the ending location?) How do you account for the difference?
4. Suppose that a third observer started in Figure 1 at (5 km, 5 km) and by the time shown in Figure 2 had moved to (3 km, 7 km). How rapidly did s/he observe temperature to change, on the average? How rapidly did the temperature change at the observer's starting location, on the average? What about the ending location? Does there seem to be a discrepancy between what the observer observes and what happens at the observer's starting and ending locations? How do you account for the differenc in sign?
5. Refer to Questions (1) and (2) above. What is the average 2-D temperature gradient in the direction of the first observer's motion, between the observer's starting and ending points (assuming the observer moved in a straight line between the two times shown)?
6. How fast did the first observer travel (on the average) during the period in question here?
7. Repeat questions 5 and 6 for the second and third observers.

8. Test the relationship below between the total and partial derivatives of temperature for each of these three observers, using finite difference estimates in place of the derivatives:

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + c\frac{\partial T}{\partial s}$$

where T is temperature (a field variable), c is the speed of the observer and s is a position coordinate along the observer's trajectory (that is, s is part of a "natural" coordinate system). If $\frac{\partial T}{\partial t}$ isn't the same at the starting and ending points, you might want to average them. Similarly, if $\frac{\partial T}{\partial s}$ isn't the same at the starting and ending times, you might want to calculate it at both times and average them. This is in the spirit of the fact that a finite difference estimate of $\frac{dT}{dt}$ represents that *average* value of $\frac{dT}{dt}$ over the finite time period, so if the relation above is to hold, we might need to average the other quantities in space and/or time correspondingly.

9. Suppose that t_0 is the time shown in Figure 1 and t_1 is the time shown in Figure 2. The s-coordinate of the first observer's position is $s_{\mathrm{obs1}}(t)$. (We write it as a function of time because the observer could be moving, and in fact is moving in this example.)

(By construction of the coordinate system, the n- and z-coordinates of the observer's position are 0 and 0, respectively. That is, the observer always lies along his/her own trajectory, which we define to be the s-axis of the natural coordinate system.)

What are each of the following temperatures?

- $T(s_{\mathrm{obs1}}(t_0),0,0,t_0)$
- $T(s_{\mathrm{obs1}}(t_1),0,0,t_1)$
- $T(s_{\mathrm{obs1}}(t_0),0,0,t_1)$
- $T(s_{\mathrm{obs1}}(t_1),0,0,t_0)$

Figure 1: A field of temperature (°C) at 11:00 pm on Fri., Sept. 13, 2015

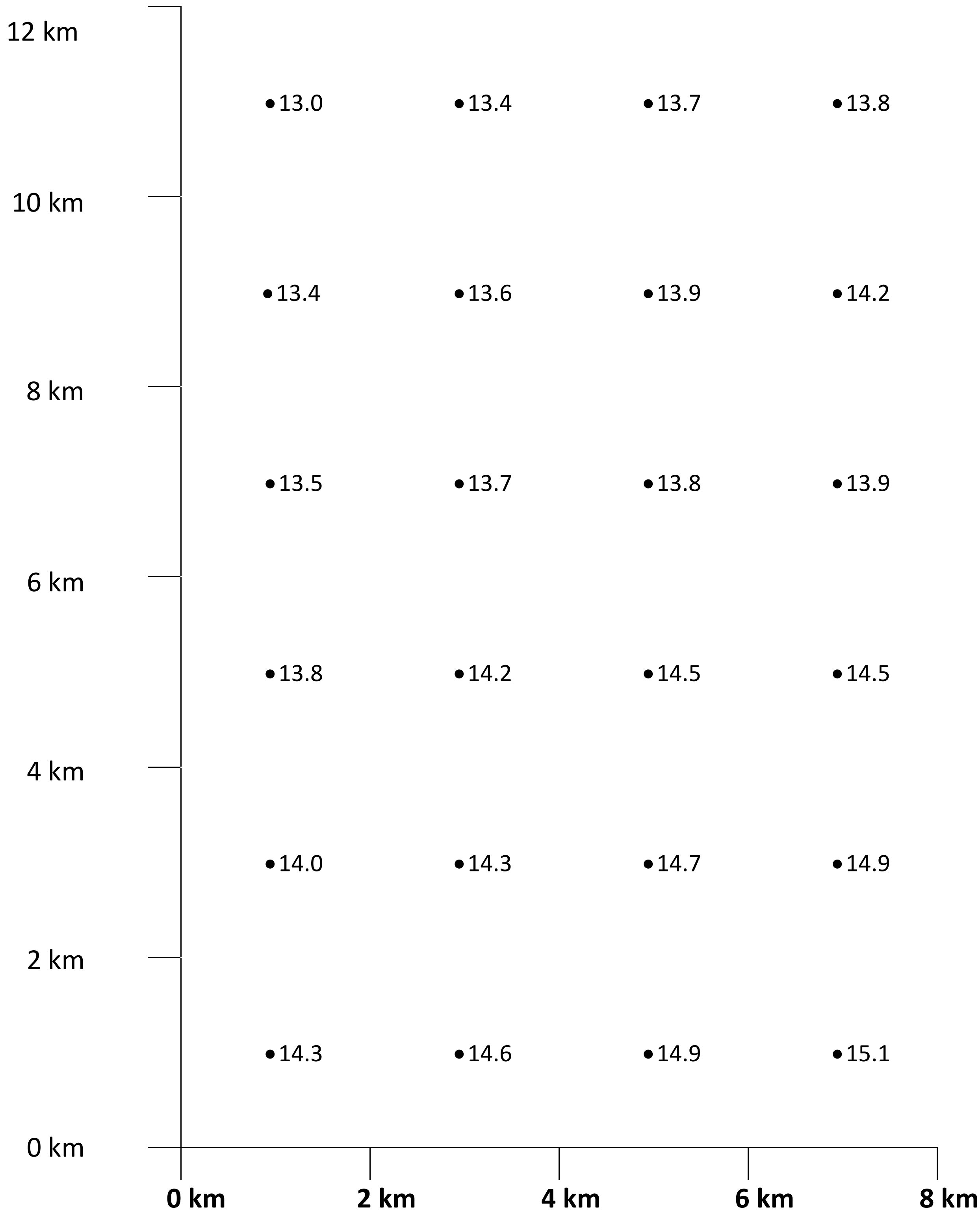

Figure 2: A field of temperature (°C) at 1:00 am on Fri., Sept. 14, 2015