

Different Types of Time Derivatives (Rates at which Some Physical Property of Matter Changes w/r/t Time)

Name	Symbol	Description/ Physical Interpretation	Finite Difference Approximations	Locations at which Observations or Measurements are Recorded over Time	Velocity of Locations at which Observations Are Recorded	Overlap between Derivatives: Special Cases	Physical Interpretation and Example Application Contexts
Total derivative	$\frac{d()}{dt}$	Rate at which an observer/instrument/probe observes or measures some field variable to change w/r/t time as the observer moves (or not) through space	$\frac{dQ}{dt} \approx \frac{Q(\vec{r}_{obs}(t + \Delta t), t + \Delta t) - Q(\vec{r}_{obs}(t), t)}{(t + \Delta t) - t}$ (where $Q(\vec{r}, t)$ is a field variable) (There are other approximations, too. In the limit as $\Delta t \rightarrow 0$, finite difference estimates become exact definitions of the derivative.)	$\vec{r}_{obs}(t)$: Location of an observer, instrument or probe that can observe or measure a physical property of matter. This location can vary with time (t).	$\vec{c}_{obs} \equiv \frac{d\vec{r}_{obs}}{dt}$ Velocity of an observer, instrument or probe that is capable of measuring some physical property		Rate of change of some physical property measured from a platform that might be moving through the field of that physical property (such as a plane or boat or car).
Local/Eulerian derivative	$\frac{\partial ()}{\partial t}$	Rate at which some field variable changes w/r/t time at a fixed location .	$\frac{\partial Q}{\partial t} \approx \frac{Q(\vec{r}, t + \Delta t) - Q(\vec{r}, t)}{(t + \Delta t) - t}$ (There are other approximations, too. In the limit as $\Delta t \rightarrow 0$, finite difference estimates become exact definitions of the derivative.)	\vec{r} : A (fixed) location in space (independent of time)	0 (the velocity of the location where observations/measurements is zero, i.e., this location isn't changing)	Special case of the total derivative when $\vec{c}_{obs} = 0$ (i.e., when the observer/instrument/probe isn't moving)	Rate of change of a physical property at a fixed location. Examples of fixed locations where this is relevant: weather station, stream gauge.
Material/parcel/Lagrangian derivative	$\frac{D()}{Dt}$	Rate at which some property of a bit of matter (i.e. an "object", e.g. a fluid parcel) changes w/r/t time.	$\frac{DQ}{Dt} \approx \frac{Q(\vec{r}_{obj}(t + \Delta t), t + \Delta t) - Q(\vec{r}_{obj}(t), t)}{(t + \Delta t) - t}$ (There are other approximations, too. In the limit as $\Delta t \rightarrow 0$, finite difference estimates become exact definitions of the derivative.)	$\vec{r}_{obj}(t)$: Location of some object (i.e., a bit of matter, such as a fluid parcel), which might vary with time (t)	$\vec{c}_{obj} \equiv \frac{D\vec{r}_{obj}}{Dt}$ Velocity of a material object, some property of which is being observed or measured. If the object is a bit of fluid (i.e., a fluid parcel), we use a distinct symbol: $\vec{U} \equiv \frac{D\vec{r}_{parcel}}{Dt}$	Special case of the total derivative when $\vec{c}_{obs} = \vec{c}_{obj}$ or $\vec{c}_{obs} = \vec{U}$ (i.e., when the observer/instrument/probe "follows" an object such as fluid parcel, recording the physical property of the same object over time)	Rate of change of a physical property of a bit of matter, "following" that bit of matter. Hard to measure in practice, but theoretically extremely important because it appears in conservation laws.