ERTH 430:
Fluid Dynamics
in Earth Systems

The Governing Equations

(Rectangular Coordinates)

Velocity Conservation Equations (Equations of Motion) (non-rotating)

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

Temperature Conservation Equation

$\frac{DT}{Dt} =$ rate of change due to radiative absorption

– rate of change due to radiative emission

+/– rate of change due to phase changes of water

+/– rate of change due to conduction

+/– rate of change due to compression/expansion associated with changes in pressure ($\propto Dp/Dt$)

Density Conservation Equation (Continuity Equation)

$$\frac{D\rho}{Dt} = -\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)$$

Equation of State

- For gases (Ideal Gas Law): $p = \rho R T$
- For liquids: Complex empirical relationship among pressure, temperature, density, and composition of the liquid (careful lab observations fitted to a set of polynomial curves)

ERTH 430:
Fluid Dynamics
in Earth Systems

The Governing Equations in Tendency Form
(Rectangular Coordinates)

<u>Velocity Tendency Equations</u>

$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial x} \quad + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - w\frac{\partial v}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial y} \quad + \frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t} = -u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z} - \frac{1}{\rho}\frac{\partial p}{\partial z} - g + \frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

<u>Temperature Tendency Equation</u>

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - w\frac{\partial T}{\partial z} +$$ rate of change due to radiative absorption

– rate of change due to radiative emission

+/– rate of change due to phase changes of water

+/– rate of change due to conduction

+/– rate of change due to compression/expansion associated with changes in pressure ($\propto Dp/Dt$)

<u>Density Tendency Equation</u> (Continuity Equation in Tendency Form)

$$\frac{\partial \rho}{\partial t} = -u\frac{\partial \rho}{\partial x} - v\frac{\partial \rho}{\partial y} - w\frac{\partial \rho}{\partial z} - \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) \text{ (advective form)}$$

$$= -\left(\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z}\right) \text{ (mass flux divergence form)}$$

<u>Equation of State</u>

- For gases (Ideal Gas Law): $p = \rho RT$
- For liquids: Complex empirical relationship among pressure, temperature, density, and composition of the liquid (careful lab observations fitted to a set of polynomial curves)