In-Class Exercise: Derivatives and Temporal Rates of Change

The items on the list of mathematical definitions and relations on the next page (numbered 1 to 11) mostly involve derivatives of some kind, many of them interpreted as temporal rates of change.

For each item on the list of statements in English below (labeled A-M), identify the one or more items on the *numbered* list of mathematical expressions that match it reasonably well, if such a match exists. Be prepared to explain the reasons for your choices.

(A) Definition of the velocity of an "observer" or a probe of some sort (possibly imaginary) through space, and its representation in terms of the components of that velocity in rectangular or Cartesian coordinates.

(B) A quantity that varies with spatial position and time, where the coordinates of spatial position can be represented in a variety of ways such that the individual coordinates are not necessarily equal to each other on a one-to-one basis.

(C) A relation between the rate at which a property of a (possibly mobile) bit of material varies with respect to time and the spatial and temporal derivatives of that variable. The variable in this case is necessarily a field variable (that is, a quantity that varies continuously with respect to spatial position and time).

(D) Definition of a derivative of a variable that depends on only one independent variable.

(E) Definition of a partial derivative of a variable that depends on more than one independent variable.

(F) A relation between the rate at which some quantity, as measured by a (possibly imaginary, possibly mobile) probe varies with respect to time and the spatial and temporal derivatives of that variable. The variable in this case is necesssarily a field variable (that is, a quantity that varies continuously with respect to spatial position and time).

(G) The representation of the spatial gradient of a quantity in terms of the components of that gradient expressed in rectangular or Cartesian coordinates.

(H) The definition of a material (Lagrangian) derivative.

(I) The definition of a local (Eulerian) derivative.

(J) The definition of a total derivative.

(K) The definition of the velocity of a fluid parcel, and its relation to the components of that velocity in rectangular or Cartesian coordinates.

(L) Definition of a derivative evaluated at a particular point.

(M) The rate of change with respect to time of the property of a (possibly moving) baseball.

Some Mathematical Definitions and Relations

(1) $$\frac{df(x)}{dx} \equiv \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}\right) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x)-f(x)}{\Delta x}\right) \equiv \lim_{x' \to x}\left(\frac{f(x')-f(x)}{x'-x}\right)$$

(2) $$\frac{df(x_0)}{dx} \equiv \left.\frac{df(x)}{dx}\right|_{x=x_0} \equiv \lim_{x \to x_0}\left(\frac{f(x)-f(x_0)}{x-x_0}\right)$$

(3) $$f(\vec{r},t) \equiv f(x,y,z,t) = f(x',y',z',t) = f(\theta,\varphi,r,t) = f(\theta,R,z,t)$$

where generally $x \neq x' \neq \theta$, $y \neq y' \neq \varphi \neq R$, and $z \neq z' \neq r$

(4)(a)

$$\frac{\partial f(x,y,z,t)}{\partial x} \equiv \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x,y,z,t)-f(x,y,z,t)}{(x+\Delta x)-x}\right) = \lim_{\Delta x \to 0}\left(\frac{f(x+\Delta x,y,z,t)-f(x,y,z,t)}{\Delta x}\right)$$

$$\frac{\partial f(x,y,z,t)}{\partial y} \equiv \lim_{\Delta y \to 0}\left(\frac{f(x,y+\Delta y,z,t)-f(x,y,z,t)}{(y+\Delta y)-y}\right) = \lim_{\Delta y \to 0}\left(\frac{f(x,y+\Delta y,z,t)-f(x,y,z,t)}{\Delta y}\right)$$

(4)(b)

$$\frac{\partial f(x,y,z,t)}{\partial t} \equiv \lim_{\Delta t \to 0}\left(\frac{f(x,y,z,t+\Delta t)-f(x,y,z,t)}{(t+\Delta t)-t}\right) = \lim_{\Delta t \to 0}\left(\frac{f(x,y,z,t+\Delta t)-f(x,y,z,t)}{\Delta t}\right)$$

(5) $$\nabla f(x,y,z,t) = \hat{\mathrm{i}}\frac{\partial f(x,y,z,t)}{\partial x} + \hat{\mathrm{j}}\frac{\partial f(x,y,z,t)}{\partial y} + \hat{\mathrm{k}}\frac{\partial f(x,y,z,t)}{\partial z}$$

(6) $$\frac{dQ(\vec{\mathrm{r}}_{\mathrm{obs}},t)}{dt} \equiv \lim_{\Delta t \to 0}\left(\frac{Q(\vec{\mathrm{r}}_{\mathrm{obs}}(t+\Delta t),t+\Delta t)-Q(\vec{\mathrm{r}}_{\mathrm{obs}}(t),t)}{(t+\Delta t)-t}\right)$$

(7) $$\frac{DQ(\vec{\mathrm{r}}_{\mathrm{obj}},t)}{Dt} \equiv \lim_{\Delta t \to 0}\left(\frac{Q(\vec{\mathrm{r}}_{\mathrm{obj}}(t+\Delta t),t+\Delta t)-Q(\vec{\mathrm{r}}_{\mathrm{obj}}(t),t)}{(t+\Delta t)-t}\right)$$

(8) $$\vec{\mathrm{c}} \equiv \frac{d\vec{\mathrm{r}}_{obs}(t)}{dt} = \hat{\mathrm{i}}c_x + \hat{\mathrm{j}}c_y + \hat{\mathrm{k}}\,c_z \equiv \hat{\mathrm{i}}\frac{dx_{obs}}{dt} + \hat{\mathrm{j}}\frac{dy_{obs}}{dt} + \hat{\mathrm{k}}\frac{dz_{obs}}{dt}$$

(9) $$\vec{\mathrm{U}} \equiv \frac{D\vec{\mathrm{r}}_{parcel}(t)}{Dt} = \hat{\mathrm{i}}\,u + \hat{\mathrm{j}}v + \hat{\mathrm{k}}\,w \equiv \hat{\mathrm{i}}\frac{Dx_{parcel}}{Dt} + \hat{\mathrm{j}}\frac{Dy_{parcel}}{Dt} + \hat{\mathrm{k}}\frac{Dz_{parcel}}{Dt}$$

(10) $$\frac{dQ(\vec{r}_{obs}(t),t)}{dt} = \frac{\partial Q(\vec{r},t)}{\partial t} + \vec{c}\cdot\nabla Q(\vec{r},t)$$

(11) $$\frac{DQ(\vec{r}_{parcel}(t),t)}{Dt} = \frac{\partial Q(\vec{r},t)}{\partial t} + \vec{U}\cdot\nabla Q(\vec{r},t)$$